SECONd IS ALWAYS FIRST

A famous set of four non-transitive dice known as 'EFRON DICE' are invented by the American Statistician Brad Efron. It is set of unusual dice.

HOW TO PLAY?
Player one selects one of the dice.
Player two selects one of the remaining three dice.
The player who casts a larger number displayed on the dice gets one point.
The one who gets more points after ten throws, wins.

These dice are non-transitive
Transitivity - If 'a' is related to 'b' and 'b' is related to 'c' implies that 'a' is related to 'c', then we say that relation is Transitive.

The dice in this set are non-transitive. As, here GREEN die beats RED one, RED beats BLUE, BLUE beats YELLOW. Now, it is expected that GREEN should beat YELLOW. But, unexpectedly YELLOW beats GREEN. So, it makes a cycle as shown in adjacent figure.

From cycle (or above observation) we note that, no die in the set is the strongest or the weakest as every die is beaten by some die from the set.
HOW ONE BEATS ANOTHER?
GREEN  3 3 3 3 3 3 RED  2 2 2 6 6
BLUE  1 1 5 5 5 YELLOw  0 0 4 4 4 4

Probability of GREEN winning over RED is 2/3
Probability of RED winning over BLUE is 2/3

Probability of BLUE winning over YELLOW is 2/3
Probability of YELLOW winning over GREEN is 2/3
We have two pairs of dice opposing each other on the circle. In fact, RED beats YELLOW with probability $\frac{5}{9}$.

![Diagram of RED beats YELLOW]

But the GREEN die and BLUE die are equal with each having 50 percent chance of winning and neither die dominating.

![Diagram of GREEN and BLUE have Equal Probabilities]

The mathematical principles used above are:

1. The ADDITION Principle: If there are $r_1$ different objects in the first set, $r_2$ different objects in the second set,......and $r_m$ different objects in the $m$th set and if the different sets are disjoint, then the number of ways to select an object from one of the $m$ sets i.e. $r_1 + r_2 + \ldots + r_m$.

2. The MULTIPLICATION Principle: Suppose a procedure can be broken into on successive stages, with $r_1$ different outcomes in the first stage, $r_2$ different outcomes in the second stage,........and $r_m$ different outcomes in the $m$th stage. If the number of outcomes at each stage is independent of the choices in the previous stages and if the composite outcomes are all distinct, then the total procedure has $r_1 \times r_2 \times \ldots \times r_m$ different composite outcomes.