MATHEMATICS

(1) The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ is/are

(a) infinitely many
(b) 0
(c) 1
(d) 3

(2) The digit in the units place of the decimal representation of $1! + 2! + \ldots + 2018!$ is

(a) 1
(b) 3
(c) 8
(d) 0

(3) If ω is an imaginary cube roots of unity then $(1-\omega^2)(1-\omega^4)(1-\omega^8)(1-\omega^{10})$ is

(a) -27
(b) -9
(c) 9
(d) 27

(4) The smallest positive integer for which $(1+i)^{2n} = (1-i)^{2n}$ is

(a) 2
(b) 4
(c) 8
(d) 16

(5) Let A= $\begin{pmatrix} 4 & 4k & k \\ 0 & k & 4k \\ 0 & 0 & 4 \end{pmatrix}$. If $\det(A^2) = 16$, then $|k|$ is

(a) 1
(b) 1/4
(c) 4
(d) 4^2

(6) The number of numbers greater than 3000 which can be formed using the digits $0, 1, 2, 3, 4, 5$ without repetition is

(a) 1240
(b) 1280
(c) 1320
(d) 1380

(7) In an examination a candidate has to pass in each of the four subjects. The number of ways in which he can fail is

(a) 15
(b) 20
(c) 25
(d) 10

(8) The common difference of an Arithmetic Progression in which $a_{18} - a_{14} = 32$ is

(a) 8
(b) -8
(c) -4
(d) 4

(9) A natural number, when increased by 12, equals 160 times its reciprocal. The number is

(a) 4
(b) 8
(c) 16
(d) 20

(10) A train travels a distance of 300 km at constant speed. If the speed of the train is increased by 5 km/hr, the journey would have taken 2 hours less. The original speed of the train is.

(a) 25 km/hr
(b) 30 km/hr
(c) 35 km/hr
(d) 40 km/hr

(11) The value of k for which the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines is

(a) 1/2
(b) - 1/2
(c) 2
(d) -2

(12) The area of a triangle with vertices A(3,0), B(7,0) and C(8,4) is

(a) 14 sq units
(b) 6 sq units
(c) 10 sq units
(d) 8 sq units

(13) If $\sin x - \cos x = 0$, then the value of $(\sin^4 x + \cos^4 x)$ is

(a) 1

(b) 1/4

(c) 3/4

(d) 1/2

(14) The area of a square that can be inscribed in a circle of radius 8 cm is

(a) 256 cm^2

(b) 128 cm^2

(c) 64 cm^2

(d) 64 $\sqrt{2}$ cm^2

(15) If the volumes of the two spheres are in the ratio 64 : 27, then the ratio of their surface areas is

(a) 3:4

(b) 4:3

(c) 9:16

(d) 16:9

(16) If A= $\begin{pmatrix} k & 0 \\ 2 & 3 \end{pmatrix}$ and I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $A^2 = 9I$ for

(a) k = 4

(b) k = 3

(c) k = -3

(d) no such k exists

(17) Consider the function

$$f(x) = \begin{cases} x^2 & x \text{ is rational, } x \neq 0 \\ -x^2 & x \text{ is irrational} \\ \text{undefined} & x = 0 \end{cases}$$

Then

(a) there is no a for which $\lim_{x \to a} f(x)$ exists

(b) there may be some a for which $\lim_{x \to a} f(x)$ exists, but it is impossible to say without more information

(c) $\lim_{x \to a} f(x)$ exists only when $a = 0$

(d) $\lim_{x \to a} f(x)$ exists for infinitely many a

(18) If a function f is not defined at $x = a$,

(a) $\lim_{x \to a} f(x)$ cannot exist

(b) $\lim_{x \to a} f(x)$ could be 0

(c) $\lim_{x \to a} f(x)$ must approach ∞

(d) none of the above.

(19) $\lim_{x \to 0} x^2 \sin(1/x)$

(a) does not exist because no matter how close x gets to 0, there are x's near 0 for which $\sin(1/x) = 1$, and some for which $\sin(1/x) = -1$

(b) does not exist because the function values oscillate around 0

(c) equals 1

(d) equals 0

(20) You know the following statement is true:

If $f(x)$ is a polynomial, then $f(x)$ is continuous.

Which of the following is also true?

(a) If $f(x)$ is not continuous, then it is not a polynomial.

(b) If $f(x)$ is continuous, then it is a polynomial.

(c) If $f(x)$ is not a polynomial, then it is not continuous.

(d) All of the above.

(21) The line tangent to the graph of $f(x) = x$ at $(0, 0)$

(a) does not exist

(b) is $y = 0$

(c) is $y = x$

(d) is not unique. There are infinitely many tangent lines.

(22) $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots$ (up to 100 terms) is

(a) $2^{-100} + 99$

(b) $2^{-100} + 100$

(c) $2^{-101} + 99$

(d) $2^{-101} + 100$

(23) If the radius of a circle increases from r_1 to r_2, then the average rate of change of the area of the circle is

(a) less than $2\pi r_2$

(b) greater than $2\pi r_1$

(c) equal to $2\pi \frac{r_1 + r_2}{2}$

(d) all of the above

(24) The derivative of $f(x) = x|x|$ at $x = 0$

(a) is 0.

(b) does not exist, because $|x|$ is not differentiable at $x = 0$

(c) does not exist, because f is defined piecewise

(d) does not exist, because the left and right hand limits do not agree.

(25) If $f'(a)$ exists, $\lim_{x \to a} f(x)$

(a) it must exist, but there is not enough information to determine it exactly

(b) equals $f(a)$

(c) equals $f'(a)$

(d) it may not exist

*****