The Soma cube is a solid dissection puzzle invented by Piet Hein in 1933 during a lecture on quantum mechanics conducted by Werner Heisenberg. Seven pieces made out of unit cubes must be assembled into a 3x3x3 cube. The pieces can also be used to make a variety of other 3D shapes.

The pieces of the Soma cube consist of all possible combinations of three or four unit cubes, joined at their faces, such that at least one inside corner is formed. There is one combination of three cubes that satisfies this condition, and six combinations of four cubes that satisfy this condition, of which two are mirror images of each other (see Chirality). Thus, $3 + (6 \times 4) = 27$ which is exactly the number in a 3 x 3 x 3 larger cube.

The Soma cube is sometimes regarded as the 3D equivalent of polyominoes. There are interesting parity properties relating to solutions of the Soma puzzle.

Soma has been discussed in detail by Martin Gardner and John Horton Conway, and the book Winning Ways for your Mathematical Plays contains a detailed analysis of the Soma cube problem. There are 240 distinct solutions of the Soma cube puzzle, excluding rotations and reflections: these are easily generated by a simple recursive backtracking search computer program similar to that used for the eight queens puzzle.

The seven Soma pieces are six polycubes of order four and one of order three:

- Piece 1, or "V".
- Piece 2, or "L": a row of three blocks with one added below the left side.
- Piece 3, or "T": a row of three blocks with one added below the center.
- Piece 4, or "Z": bent triomino with block placed on outside of clockwise side.
- Piece 5, or "A": unit cube placed on top of clockwise side. Chiral in 3D.
- Piece 6, or "B": unit cube placed on top of anticlockwise side. Chiral in 3D.
- Piece 7, or "P": unit cube placed on bend. Not chiral in 3D.
Some facts regarding the Solution

In each of the 240 solutions to the cube puzzle, there is only one place that the "T" piece can be placed. Each solved cube can be rotated such that the "T" piece is on the bottom with its long edge along the front and the "tongue" of the "T" in the bottom center cube (this is the normalized position of the large cube). This can be proven as follows: If you consider all the possible ways that the "T" piece can be placed in the large cube (without regard to any of the other pieces), it will be seen that it will always fill either two corners of the large cube or zero corners. There is no way to orient the "T" piece such that it fills only one corner of the large cube. The "L" piece can be oriented such that it fills two corners, or one corner, or zero corners. Each of the other five pieces have no orientation that fills two corners; they can fill either one corner or zero corners. Therefore, if you exclude the "T" piece, the maximum number of corners that can be filled by the remaining six pieces is seven (one corner each for five pieces, plus two corners for the "L" piece). A cube has eight corners. But the "T" piece cannot be oriented to fill just that one remaining corner, and orienting it such that it fills zero corners will obviously not make a cube. Therefore, the "T" must always fill two corners, and there is only one orientation (discounting rotations) in which it does that. It also follows from this that in all solutions, five of the remaining six pieces will fill their maximum number of corners and one piece will fill one less than its maximum (this is called the deficient piece).