Königsberg bridges

There is a river in the city names Königsberg and there are two islands in it. These two islands and bank of river were connected by seven bridges. This can be seen in the following figure:

Some citizens used take a walk in the evening along the bridges. They found that they were not able to walk over all the seven bridges crossing over each of them only once. In order to understand the reason they posed the problem before the famous mathematician, L. Euler. He proved that in this particular case, it is impossible to plan such a walk so as to cover every bridge once only. Euler’s solution became the starting point of the graph theory.

Question
If any one of the seven bridges such that the span of all the remaining six bridges is covered only once?

Question
In case an additional bridges constructed joining land to land, will it be possible to plan a walk such the all the eight bridges are crossed only once?
There are four areas of the town—on the mainland north of the river, on the mainland south of the river, on the two islands; let us label them A, B, C and D.

To “visit each part of the town” the person taking the walk should visit the points A, B, C and D. And they should cross the each bridge p, q, r, s, t, u and v just once.

And we can further simplify it to this:

So instead of taking long walks through the town, we can now just draw lines with a pencil.

Question Can you draw each line p, q, r, s, t, u and v only once, without removing your pencil from the paper (you may start at any point)?
Let’s take a step back and try some simpler shapes.  
Try these (remember: draw all the lines, but never go over any line more than once, and don’t remove your pencil from the paper.)

So, how can we know which ones work and which ones do not?
Let’s investigate!!! But first, time to learn some special words:

- A point is called a vertex (Plural vertices).
- A line is called an edge.
- The whole diagram is called a graph.

The number of edges that lead to a vertex is called the degree.

A route around a graph that visits every vertex once is called a Simple path.
A route around a graph that visits every edge once is called an Euler path.

Ok, In our bridges problem; Imagining that the lines are bridges, and you cross them once only, you have solved the puzzle, So what we want is an Euler Path and here is a clue to help you: We can tell which graphs have an Euler Path by counting how many vertices have an odd degree.

This is because of Euler. Euler showed that in any graph the number of odd vertices is always even. He also showed that a graph or a network can be drawn in a single circuit, (i.e. without lifting the pencil off the paper or retracing any line) if there are at most 2 odd vertex.

Thus, the number of the odd nodes should be 0 or 2, as it has to be even. Now lets go back to Königsberg bridge question:
Vertices A, B and have degree 3 and vertex C have degree 5, So this graph has 4 vertices of odd degree. Therefore, it does not have an Euler Path.

We have solved the Königsberg bridge question just like Euler did nearly 300 years ago!!!